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Effects of Compression and Expansion on Turbulence Intensity in Supersonic Boundary Layers

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The effects of bulk expansion and compression on turbulence levels in supersonic boundary layers were predicted using the law of angular momentum conservation. For the compression case the present analysis predicted the amplification of turbulence level quite well when the effect of extra strain rate caused by the streamline curvature is small, where it was estimated that 90–95% of the increase in the turbulence level is caused by the bulk compression. When extra strain rate effects are present, however, the contribution of the bulk compression on the total turbulence level was reduced to about 85–90%. The predicted general trend of turbulence amplification with increasing flow-turning angle, which seems to be closely related to the linear increase in the mean density, agrees well with that of other researchers. For the expansion case, however, a quantitative comparison was challenging because the available experimental data were taken too close to the end of the perturbation region, thus not showing the relaxation behavior. In both compression and expansion flows the bulk dilatation was found to be the dominant factor responsible for the changed turbulence intensity.

Nomenclature

A	=	cross-sectional area of a vortex tube
I	=	moment of inertia of a vortex tube
$k_A = \surd(S_2/S_1)$	=	contraction or expansion ratio of the flow
$k_b = k_A^2$	=	ratio of radii of the spanwise vortex
$k_n = (\rho_1/\rho_2)^{2/3}$	=	ratio of radii of the normal vortex
L	=	length of a vortex tube
L_s	=	streamwise length scale of a vortex tube
M	=	Mach number
m	=	mass of a vortex tube
R	=	radius of a vortex tube, radius of curvature in the perturbation region
S	=	cross-sectional area of the flow boundary
V	=	mean velocity
γ	=	specific heat ratio
δ	=	boundary-layer thickness
ρ	=	density
σ	=	turbulence intensity
Ω	=	vorticity

Subscripts

b	=	spanwise direction
e	=	boundary-layer edge
n	=	normal direction to the wall
s	=	streamwise direction (Fig. 2)
1	=	conditions at an upstream location
2	=	conditions at a downstream location

Superscripts

(1)	=	conditions at an upstream location
(2)	=	conditions at a downstream location

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I. Introduction

IT is well known that a bulk expansion reduces whereas bulk compression increases turbulence level.[1,2] The available experimental data in supersonic boundary layers confirm this phenomenon for bulk expansion[3–6] and bulk compression.[7–12] These researchers have found that, in compression flows with a corner or a curved wall, skin friction, turbulence levels, and turbulent mixing length increase. The results of Jayaram et al.[7] show that the amplification of turbulence level through compression is higher in the curved wall flow than in the corner flow as a result of the effects of streamwise curvature. The effects of streamline curvature were also explored by Fernando and Smits[10] and Smith and Smits.[12] They reported a similar effect. The results of Jayaram et al.[7] and Smits and Muck[13] show that shock-wave oscillation becomes an add-on amplification mechanism of turbulence level when the strength of the shock is sufficiently strong. Except for the effects of streamline curvature and shock wave, the amplification of turbulence level through compression seems to be explained by the total increase in the mean density.

For the expansion flow with a corner, Smith and Smits[4] showed by using rapid distortion analysis (RDA) that 90% of reduction in streamwise Reynolds stress was attributed to the bulk dilatation. An interesting phenomenon is that the turbulence level after the perturbation does not show any sign of relaxation even at a relatively far downstream location of $10\delta_0$, (Refs. 14–16), where δ_0 is the upstream boundary-layer thickness. In the expansion flow the effects of curvature are not well documented.

The RDA, which was developed for incompressible flow, seems to predict the variation of turbulence statistics well for supersonic boundary layers[4] and for compressible shear layers.[17] Recently, the phenomenon has also been investigated by using direct numerical simulation in shear layers.[17] Although the use of RDA in predicting turbulence evolution through compression or expansion has been successful, the RDA cannot clearly show the relative effects of dilatation and vortex stretching. According to Smith and Smits,[18] the RDA is only good for predicting the initial changes of Reynolds stresses and not valid when the distortion/perturbation is strong because of the nonlinear nature of the flow.

Kim and Samimy[19] used an alternative approach to explore the effects of expansion and vortex stretching on turbulence intensity of a supersonic boundary layer. They derived an equation, which was from the vorticity transport equation, and could explain the relative effects of vortex stretching and dilatation. For a streamwise vortex, which experiences a bulk expansion, the vorticity decreases mainly as a result of bulk dilatation although the vortex stretching by the flow acceleration increases the streamwise vorticity. The purpose of the present research is to predict the effects of bulk compression and expansion on turbulence intensity and anisotropy by using the conservation law of angular momentum. To the authors' best knowledge, this study is the very first attempt to predict the quantitative variation of turbulence level by using the conservation of angular momentum of a vortex. Although Dussauge and Gaviglio[3] and Donovan et al.[11] suggested that the variation of turbulence level through the perturbation can be explained using the conservation of angular momentum, no quantitative comparison was attempted.

II. Analysis

Two approaches could be taken: one is the use of the vorticity transport equation in quasi-two-dimensional boundary-layer flow as was used by Kim,[20] and the other is the use of the conservation of angular momentum of a vortex.[21] The former approach is more involved than the latter, but it can calculate the effect of baroclinic torque, which occurs only when the density gradient is not parallel to the pressure gradient. On the other hand, the latter analysis cannot estimate the effect of baroclinic torque because it only uses the upstream and downstream conditions. For this reason the latter approach is used in the present analysis. As in the RDA, the present analysis does not take into account the effects of viscous diffusion and dissipation.

A. Variation of Vorticity Through Perturbation

A simple analysis to determine the effect of expansion/compression was adopted for quasi-two-dimensional flows by using the conservation of angular momentum. The disadvantage of this approach, when it is compared to the other approach just mentioned, is that it cannot estimate the effects of baroclinic torque. However, this approach provides clearer insight into the compression or expansion effects on vorticity because of its simplicity and straightforward assumptions.

1. Streamwise Vortex Tube

For a streamwise vortex filament in a quasi-two-dimensional supersonic flow, as shown in Fig. 1, the vortex radius R_2 and the streamwise vorticity $\Omega_s^{(2)}$ after expansion/compression can be calculated by using the angular momentum conservation law:

$$I_1\Omega_s^{(1)} = I_2\Omega_s^{(2)}$$

Because $I = \frac{1}{2}mR^2$, one may have

$$R_1^2\Omega_s^{(1)} = R_2^2\Omega_s^{(2)} \tag{1}$$

The schematic of a two-dimensional boundary layer, which experiences expansion, is shown in Fig. 2, where the coordinate system used in the present analysis is also depicted. Once the ratio of radii R_2/R_1 is known, the vorticity change is obtained. To estimate R_2/R_1, the square root of the ratio of the cross-sectional areas of the flow boundary is defined as $k_A : k_A = \sqrt{(S_2/S_1)}$. For the streamwise vortex the ratio of cross-sectional areas of the vortex filament is approximated as that of flow boundary cross-sectional areas:

$$S_2/S_1 \approx A_2/A_1 = \pi R_2^2 \big/ \pi R_1^2 \tag{2}$$

From Eqs. (1) and (2) one can have

$$k_A = \sqrt{S_2/S_1} \approx R_2/R_1 \tag{3}$$

Now one can calculate the variation of streamwise vorticity through perturbation from Eqs. (1) and (3) as

$$\Omega_s^{(2)} = \Omega_s^{(1)} \big/ k_A^2 \tag{4}$$

In an actual flow one can alternatively calculate k_A from the isentropic relation between flow boundary cross-sectional area S and the nozzle throat area S^*:

$$\frac{S}{S^*} = \frac{1}{M}\left\{\frac{1+[(\gamma-1)/2]M^2}{1+[(\gamma-1)/2]}\right\}^{(\gamma+1)/2(\gamma-1)}$$

Because the nozzle throat area S^* is constant, one can show that

$$k_A = \sqrt{\frac{S_2}{S_1}} = \sqrt{\frac{M_1}{M_2}}\left\{\frac{1+[(\gamma-1)/2]M_2^2}{1+[(\gamma-1)/2]M_1^2}\right\}^{(\gamma+1)/4(\gamma-1)} \tag{5}$$

By using the mass conservation law $(\rho S V_s)_1 = (\rho S V_s)_2$, Eq. (4) is reduced to

$$\Omega_s = (\text{const})\rho V_s \tag{6}$$

which is the same equation reduced from the vorticity transport equation by Kim and Samimy.[19] In fact, Eqs. (4) and (6) indicate that the circulation of a vortex is conserved when there are no vorticity sources. Contrary to the incompressible counterpart, the vortex stretching by expansion in a supersonic flow results in the decrease of vorticity as proven by Eqs. (4) and/or (6).

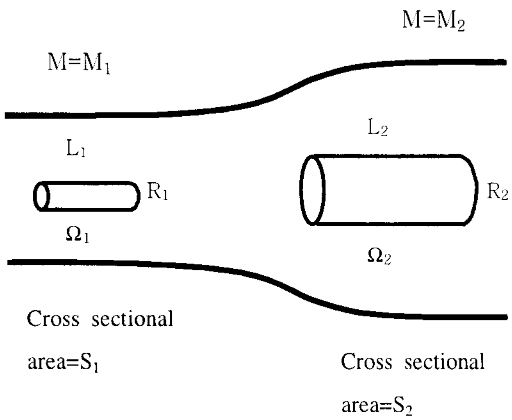

Fig. 1 Schematic of a streamwise vortex experiencing vortex stretching through expansion in a two-dimensional flow. The spanwise dimension is kept unchanged as in a two-dimensional converging–diverging nozzle.

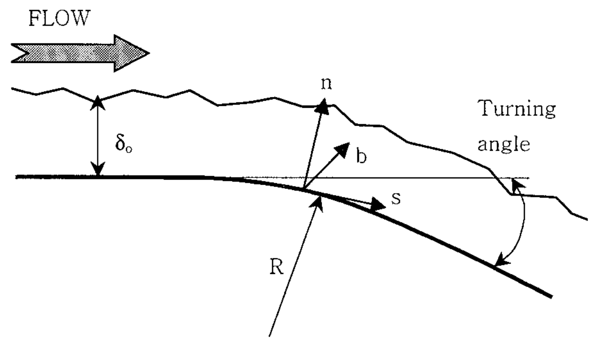

Fig. 2 Schematic of a quasi-two-dimensional boundary layer with the coordinate system used. In this figure *R* is the radius of curvature in the perturbation region.

2. Spanwise Vortex Tube

To calculate vorticity variations through dilatation, one should calculate the ratio of the radius of a vortex at a downstream location to that at an upstream location. Once the ratio is known, one can evaluate vorticity ratio from the angular momentum conservation law as in Eq. (4). One can assume that the streamwise length scale L_s of a vortex tube is proportional to the streamwise velocity scale V_s. By using the mass conservation relation $(\rho S V_s)_1 = (\rho S V_s)_2$, one obtains

$$L_s^{(2)} \big/ L_s^{(1)} \approx V_s^{(2)} \big/ V_s^{(1)} = \rho_1 S_1 / \rho_2 S_2 \tag{7}$$

Considering the mass conservation of the spanwise vortex, one can have

$$\rho_1 \pi R_1^2 L_1 = \rho_2 \pi R_s^{(2)} R_n^{(2)} L_2$$

$$R_n^{(2)} \big/ R_1 = (\rho_1/\rho_2)\big(R_1 \big/ R_s^{(2)}\big)(L_1/L_2)$$

Because the spanwise dimension of the flow boundary is fixed, one can assume $L_1/L_2 \approx 1$. In additon, one can assume $R_1/R_s^{(2)} \approx L_s^{(1)}/L_s^{(2)}$. Then, the radius ratio of the spanwise vortex for vorticity calculation is

$$R_n^{(2)} \big/ R_1 \approx S_2/S_1 = k_A^2 \equiv k_b \tag{8}$$

One can define k_b as $R_s^{(2)}/R_1$, but it was found later that Eq. (8) is more appropriate for the prediction of vorticity variations, when the measured turbulence intensity was compared with the predicted values. Finally, the spanwise vorticity at a downstream location is calculated as

$$\Omega_b^{(2)} = \Omega_b^{(1)} \big/ k_b^2 \tag{9}$$

3. Normal Vortex Tube

As for the spanwise vortex, the equation for mass conservation of the normal vortex is

$$\rho_1 \pi R_1^2 L_1 = \rho_2 \pi R_s^{(2)} R_b^{(2)} L_2$$

Then, the radius ratio in streamwise direction is

$$R_s^{(2)} \big/ R_1 = (\rho_1/\rho_2)\big(R_1 \big/ R_b^{(2)}\big)(L_1/L_2)$$

One can set $R_1/R_b^{(2)} = 1$, because the spanwise dimension of the flow boundary is fixed. When a cubical fluid element with a dimension L experiences a uniform dilatation, one can have $\rho_1 L_1^3 = \rho_2 L_2^3$ from the mass conservation law. Thus, it can be assumed that the ratio of vortex lengths is inversely proportional to one-third power of the density ratio, i.e., $L_1/L_2 = (\rho_2/\rho_1)^{1/3}$. Although the normal vortex may not be uniformly deformed, the use of $L_1/L_2 = (\rho_2/\rho_1)^{1/3}$ as the ratio of vortex lengths appeared to be good enough for the normal vortex. By using these assumptions, the radius ratio of the normal vortex for the vorticity calculation is

$$R_s^{(2)} \big/ R_1 = (\rho_1/\rho_2)^{\frac{2}{3}} \equiv k_n \tag{10}$$

Then the vorticity at a downstream location of the perturbation region is

$$\Omega_n^{(2)} = \Omega_n^{(1)} \big/ k_n^2 \tag{11}$$

One can set $k_n = R_b^{(2)}/R_1$, but it was found that Eq. (11) is more appropriate.

Equations (3), (4), and (8–11) will be used to calculate the variation of turbulence through perturbation. Equations (4) and (6) show how the density change through expansion/compression plays an important role in the vorticity transport as discussed by Kim et al.[22] and Kim and Samimy.[19]

B. Turbulence Intensity Variation

It has been known since the 1920s that a turbulent flow contains eddies of ever-smaller sizes[23] and is characterized by a high level of fluctuating vorticity.[24,25] Thus, vortex dynamics plays an essential role in the evolution of turbulent flows.[24] These provide sufficient grounds for using Eqs. (3), (4), and (8–11) to calculate turbulence intensity variations through expansion or compression. In this analysis it is assumed that vortex tubes stay at the same relative normal position in the boundary layer after they pass through the perturbation region.

To calculate turbulence level/intensity variations through expansion or compression, let σ_s, σ_b, and σ_n be the rms values of the velocity fluctuations in the streamwise, spanwise, and normal directions, respectively. The velocity fluctuation in each direction is generated by the other two vortices aligned in the other two directions. For example, the streamwise velocity fluctuation is generated by both spanwise and normal vortices. Thus, streamwise, spanwise, and normal rms fluctuations are written as

$$\sigma_s^{(1)} = \sqrt{\big[(\Omega_1 R_1)_b^2 + (\Omega_1 R_1)_n^2\big]\big/2} \tag{12}$$

$$\sigma_b^{(1)} = \sqrt{\big[(\Omega_1 R_1)_n^2 + (\Omega_1 R_1)_s^2\big]\big/2} \tag{13}$$

$$\sigma_n^{(1)} = \sqrt{\big[(\Omega_1 R_1)_s^2 + (\Omega_1 R_1)_b^2\big]\big/2} \tag{14}$$

If one can assume $(\Omega_1)_s = (\Omega_1)_b = (\Omega_1)_n = \Omega_1$ and $(R_1)_s = (R_1)_b = (R_1)_n = R_1$ as in an isotropic flow, then one can have

$$\sigma_s^{(1)} = \sigma_b^{(1)} = \sigma_n^{(1)} = \Omega_1 R_1$$

The streamwise turbulence intensity/level after expansion or compression $\sigma_s^{(2)}$ is

$$\begin{aligned}\sigma_s^{(2)} &= \sqrt{\frac{(\Omega_2 R_2)_b^2 + (\Omega_2 R_2)_n^2}{2}} \\ &= \sqrt{\frac{\big[\big(\Omega_1/k_b^2\big)k_b R_1\big]^2 + \big[\big(\Omega_1/k_n^2\big)k_n R_1\big]^2}{2}} \\ &= \Omega_1 R_1 \sqrt{\frac{1/k_b^2 + 1/k_n^2}{2}} = \sigma_s^{(1)} \sqrt{\frac{1}{2}\left(\frac{1}{k_b^2} + \frac{1}{k_n^2}\right)}\end{aligned} \tag{15}$$

In the same way, spanwise and normal turbulence intensity/levels after expansion or compression $\sigma_b^{(2)}$ and $\sigma_n^{(2)}$ are

$$\begin{aligned}\sigma_b^{(2)} &= \sqrt{\frac{(\Omega_2 R_2)_n^2 + (\Omega_2 R_2)_s^2}{2}} \\ &= \Omega_1 R_1 \sqrt{\frac{1/k_n^2 + 1/k_A^2}{2}} = \sigma_b^{(1)} \sqrt{\frac{1}{2}\left(\frac{1}{k_n^2} + \frac{1}{k_A^2}\right)}\end{aligned} \tag{16}$$

$$\sigma_n^{(2)} = \sqrt{\frac{(\Omega_2 R_2)_s^2 + (\Omega_2 R_2)_b^2}{2}} = \sigma_n^{(1)} \sqrt{\frac{1}{2}\left(\frac{1}{k_A^2} + \frac{1}{k_b^2}\right)} \tag{17}$$

For compression and expansion flows k_A, k_b, and k_n are less and greater than one, respectively. For an example, k_A and k_b in a flow experiencing compression are less than one because the cross-sectional area decreases, which can be estimated from Eqs. (3) and (8). The value of k_n is also less than one because of the increased mean density as can be estimated from Eq. (10). Thus, the vorticity of a vortex that experiences compression increases, while that of a vortex in expansion flows decreases as shown in Fig. 3. In the figure the upstream Mach number is 2.9 in both compression and expansion flows. In both flows the relative variation of vorticity in streamwise direction is less than that in other directions.

From these results it is clear that expansion or favorable pressure gradient decreases turbulence intensity and vorticity, while compression increases them. By using Eqs. (15–17), the ratio of the turbulence level/intensity after the perturbation to that before the perturbation can be calculated. The results of such calculations are shown in Table 1. In Eqs. (15–17), the effects of bulk dilatation and vortex stretching/compression were taken into account. The effects of bulk dilatation dominate that of the vortex stretching/compression. The flow conditions at a downstream location were calculated by using the isentropic and oblique-shock relations for expansion and compression flows, respectively. When calculating Mach number in the middle of boundary layer, it was assumed that the Mach number in the middle of boundary layer is 88% of that at the boundary layer edge, i.e.,

$$M|_{n=\delta/2} = 0.88 M_e \tag{18}$$

The value 0.88 was taken from the Van Driest's velocity profile for compressible flows. In addition, it was assumed that the ratio of density after the perturbation to that before the perturbation is equal to the ratio at the boundary-layer edge. The calculated amplification

Table 1 Comparison of the predicted and measured data on changes of turbulence intensity through compression and expansion in a supersonic boundary layer

Reference	M_∞	Type of flow	Turning angle, deg[a]	Raw data type	Converted data, $\sigma^{(2)}/\sigma^{(1)}$	Prediction, $\sigma^{(2)}/\sigma^{(1)}$	Deviation, %
Donovan et al.[11]	2.86	Cmp	16, $R=12.5\delta_0$[b]	$(\rho\overline{u'^2})/(\rho U^2)_{ref}$	1.88	1.71	−9.4
				$(\rho\overline{v'^2})/(\rho U^2)_{ref}$	1.82	1.58	−13.0
Fernando and Smits[10]	2.92	Cmp	8, oblique shock,	$\overline{u'^2}/U^2_{ref}$	1.36	1.36	−0.1
			on flat plate	$\overline{v'^2}/U^2_{ref}$	1.25	1.30	4.5
Jayaram et al.[7]	2.87	Cmp	8, $R=10\delta_0$	$(\rho\overline{u'^2})/(\rho U^2)_{ref}$	1.53	1.35	−11.6
			8, $R=50\delta_0$		1.56		−13.2
			8, $R=0$		1.36		−0.2
Smith and Smits[12]	2.86	Cmp	16, oblique shock, on flat plate	$(\rho\overline{u'^2})/(\rho U^2)_{ref}$	1.85	1.70	−7.6
Arnette et al.[6]	3.0	Exp	7, $R=0$	u'_{rms}/U_{ref}	0.621	0.727	16.9
			7, $R=50\delta_0$		0.669	0.727	8.6
			14, $R=0$		0.502	0.509	1.4
			14, $R=50\delta_0$		0.536	0.509	−5.1
Dussauge and Gaviglio[3]	1.76	Exp	12, $R=0$	$\overline{u'^2}/U^2_{ref}$	0.831	0.714	−14.1
Smith and Smits[4]	2.84	Exp	20, $R=0$	$\overline{u'^2}/U^2_{ref}$	0.616	0.383	−37.8

[a]For the flat-plate data the turning angle is an equivalent angle based on the pressure rise across the shock waves.
[b]$R=$ radius of curvature in the perturbation region.

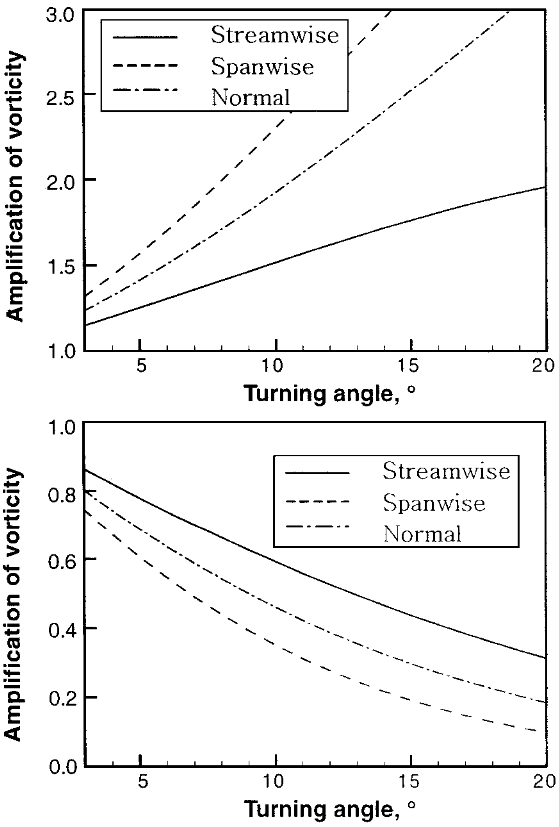

Fig. 3 Amplification of vorticity of a vortex that experiences dilatational perturbation: a) compression and b) expansion. In both cases the upstream Mach number is 2.9.

factors for streamwise and normal velocity fluctuations $\sigma_s^{(2)}/\sigma_s^{(1)}$ and $\sigma_n^{(2)}/\sigma_n^{(1)}$ are shown in Figs. 4 and 5 along with the anisotropy $\sigma_n^{(2)}/\sigma_s^{(2)}$ and density ratios.

C. Comparison with Experimental Data

Because most of the available experimental data is in the form of mass flux fluctuation, it was converted into velocity fluctuation or rms of velocity. As mentioned before, this study is the very first attempt, to the author's best knowledge, to predict the quantitative variation of turbulence level by using the conservation of angular momentum of a vortex.

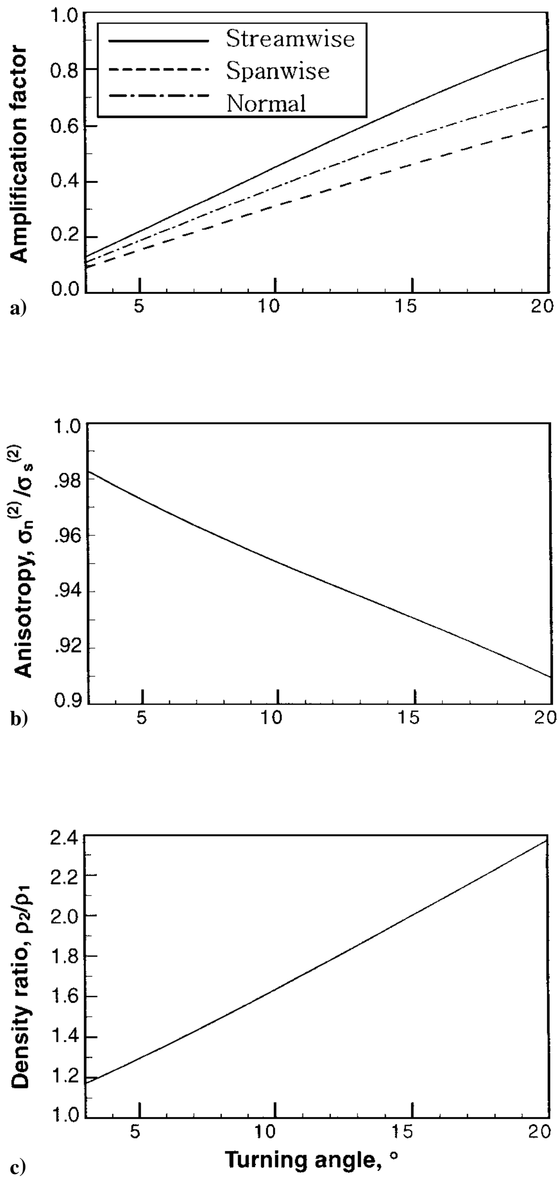

Fig. 4 Variation of a) turbulence amplification factor, b) anisotropy, and c) density ratio through a compression in a concave surface or corner. The upstream Mach number is 2.9.

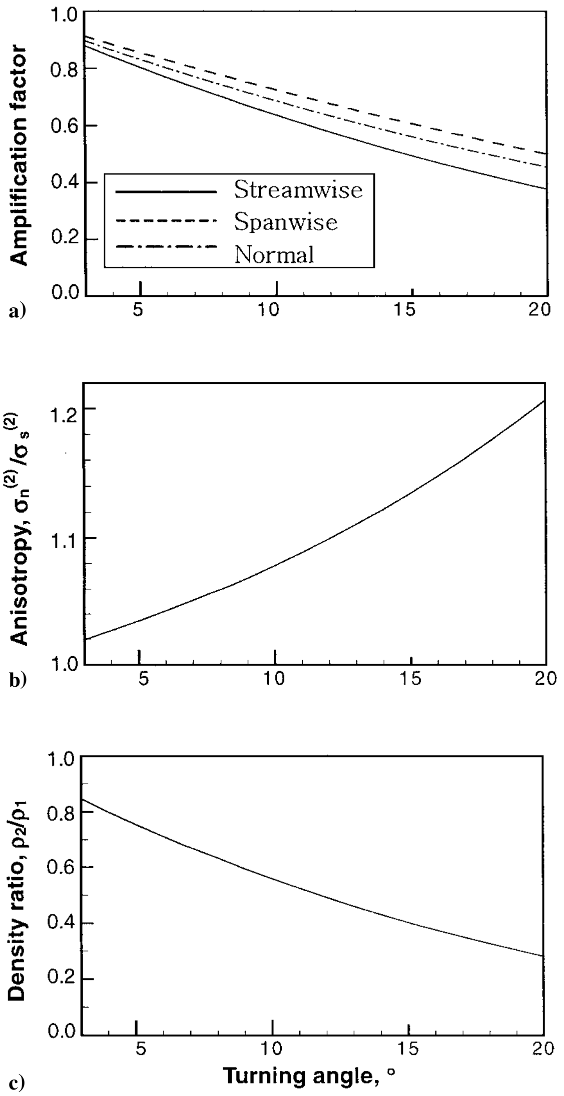

Fig. 5 Variation of a) turbulence amplification factor, b) anisotropy, and c) density ratio through a centered or gradual expansion. The upstream Mach number is 2.9.

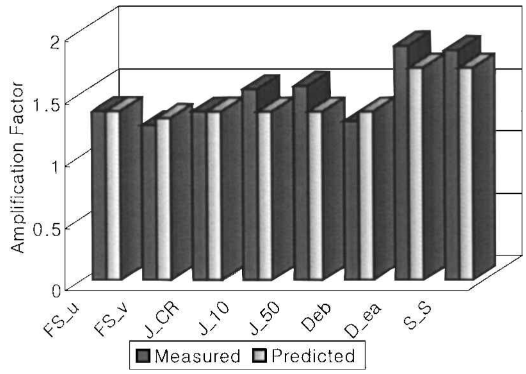

Fig. 6 Comparison of the predicted amplification factor with experimental results for compression flows (FS_u: Fernando and Smits,[10] compression by an oblique shock with an equivalent turning angle 8-deg, streamwise rms; FS_v: Fernando and Smits, with the same conditions as FS_u, normal rms; J_CR: Jayaram et al.[7] for an 8-deg compression corner flow; J_10: Jayaram et al. for an 8-deg compression flow with the streamline curvature $10\delta_0$; J_50: Jayaram et al. for 8-deg compression flow with the streamline curvature $50\delta_0$; Deb: Debieve et al.[28] for an 8-deg compression corner flow, RDA results; D_ea: Donovan et al.[11] for a 16-deg compression flow with the streamline curvature $12.5\delta_0$; S-S: Smith and Smits[12] for a compression flow by an oblique shock wave with an equivalent turning angle of 16-deg).

1. Compression Flows

All turbulence data from the middle of the boundary layer were used because viscosity effects are relatively small at this location. When turbulence data were available at multiple downstream locations, the data at a location where turbulence values just began to converge were used. As discussed by Arnette et al.,[6] large-scale structures respond to the imposed perturbation or extra strain rates slower than small-scale structures do, and thus it takes longer time for them to fully respond to the strain rates. This is the reason behind the selection of turbulence data at a downstream location where it began to converge. All experimental results show that turbulence levels begin to converge at a farther downstream location than the end of the perturbation region. The strong Reynolds analogy (SRA) proposed by Morkovin[26] was used to convert mass fluctuations data obtained using hot wire into velocity fluctuations. A detailed discussion on SRA is available in Spina et al.[27]

The predicted trend of streamwise turbulence amplification, shown in Fig. 4a, compares well with that predicted by Debieve et al.,[28] who used the RDA concept, and with that measured by Selig et al.[9] The almost linear increase in amplification of turbulence level is closely related to the linear increase in the mean density as shown in Fig. 4c. However, Smits and Muck[13] suggested that the maximum turbulence amplification is approximately proportional to the overall static pressure rise through a shock/boundary-layer interaction. When the almost-linear turbulence amplification data were compared with either the pressure or density ratio across the perturbation region, the mean density rise is more closely related to the linear amplification of turbulence intensity.

As shown in Table 1 and Fig. 6, the agreement between the predicted and measured values is quite good for the compression flows with corner or curved wall. The difference between the predicted and measured data of Jayaram et al.[7] and Donovan et al.[11] for curved-wall cases is about 10–14%. The larger difference could be partially caused by the effects of the streamline curvature and/or shock wave, which acted as an extra strain rate. According to Donovan et al.,[11] the amplification of the streamwise turbulence intensity is higher in curved flow than in the corresponding adverse-pressure-gradient flow generated by an oblique shock. When the influence of streamline curvature and shock wave is small, as in the cases of Fernando and Smits[10] and Jayaram et al.'s[7] 8-deg ramp flow, the agreement between predicted and measured data is excellent considering that the uncertainty of the measured data with a constant temperature hot wire is about ±20% in $\overline{u'^2}$ and −21% to +18% in $\overline{v'^2}$ (Ref. 10).

In general, the present method predicts turbulence intensity variations through bulk compression fairly well. This good agreement strongly suggests that the density increase through compression is the dominant factor for the increased turbulence, because the present method only takes into account the effects of bulk compression and vortex stretching, but the effects of bulk compression are dominant over that of the vortex stretching/compression. When the data of Fernando and Smits[10] and Jayaram et al.'s[7] 8-deg ramp flow, in which there is no curvature effect, are compared with the present prediction, the effect of bulk compression on turbulence could be about 90–95% of the total turbulence variation. The effect of the bulk compression on turbulence is reduced to 85–90% of the total turbulence variation when streamline curvature and/or shock wave is present. Thus, the bulk compression is the dominant factor of the increased turbulence level in a flow experiencing compression.

2. Expansion Flows

For the expansion case comparison of the measured and predicted data was challenging. This is partially because some available data, for example, the data of Dussauge and Gaviglio,[3] Smith and Smits,[4] and Arnette et al.'s[6] for 14-deg expansion flows, were not acquired sufficiently farther downstream for the flow to achieve fully relaxed conditions. Only usable data, which show relaxation behavior, are that of Arnette et al. for 7-deg expansion flows. Arnette et al.[6] showed that large-scale structures survive the perturbation and eventually respond to the perturbation at a much farther downstream location from the end of perturbation. Although the relative difference between the predicted and measured values is a little larger than that for the compression flow, the present method predicts turbulence level variation fairly well. As far as Arnette et al.'s experimental data are concerned, there was a difficulty in deciding boundary-layer thickness caused by not-so-smooth turbulence levels around the edge of the boundary layer. This probably contributed to the larger differences between the predicted and measured data.

The flow passing through successive compression and expansion showed that turbulence levels continued to decrease without displaying any relaxation even at the last available downstream location of $10\delta_0$ (Refs. 14–16). The turbulence levels at this location were lower than those in an unperturbed upstream location. This could be either caused by a second-order response of the underdamped system[29] or caused by overall destruction of the turbulence production mechanisms.[16] These findings partially explain why the measured turbulence level after relaxation showed lower values than the predicted values by the present method. In spite of the difficulties discussed, the available experimental[5,6] and predicted data seem to suggest that 80–85% of the reduction in turbulence level is caused by the mean density decrease through the expansion. Therefore, in the expansion flow also, the bulk expansion is the major factor responsible for the decreased turbulence level.

3. Anisotropy

As shown in Figs. 4b and 5b, the predicted anisotropy $\sigma_n^{(2)}/\sigma_s^{(2)}$ of turbulence shows decrease and increase with increasing deflection/turning angles for compression and expansion flows, respectively. If one recalls the different amplification of vorticity shown in Fig. 3, the variation of anisotropy is expected. For an example, the greater amplification of vorticity in spanwise and normal vortices in compression flows results in a greater amplification of streamwise velocity fluctuation as can be estimated from Eqs. (15–17) and as shown in Fig. 3a. This greater increase in the streamwise velocity fluctuation relative to those in other directions is most probably responsible for the decreased anisotropy in compression flows. If there were no vortex stretching in streamwise and normal vortices, the amplification of vorticity and thus velocity fluctuation would be the same regardless of the vortex alignment. Thus, the vortex stretching is most likely responsible for the changed anisotropy through perturbations.

For the compression flow the measured anisotropy by Ardonceau et al.[30] decreased as the present method predicts. However, the RDA data of Debieve et al.[28] show an increase of anisotropy through a compression perturbation, whereas the experimental results of Donovan et al.[11] and Fernando and Smits[10] show no change in anisotropy. Because of the different trend of available data for anisotropy, a quantitative comparison was not attempted.

For the expansion flow the predicted trend of anisotropy increase through expansion matches with that of experimental results of Arnette.[5] Here again, a quantitative comparison was not carried out as a result of the difficulty in determining the boundary-layer thickness.

III. Conclusions

An attempt was made to explore the effects of bulk expansion and compression on turbulence levels in supersonic boundary layers. Although two approaches of using the mean vorticity transport equation and the law of angular momentum conservation are available, the latter approach is used in the present analysis. For a streamwise vortex tube the vorticity variation through compression or expansion was dominated by the density change rather than by vortex stretching or compression. For the compression flow the present analysis predicted the amplification of turbulence level quite well when there were negligible effects of extra strain rate caused by streamline curvature and shock wave. In the case of negligible effects of streamline curvature and shock wave, it was estimated that 90–95% of the increase in the total turbulence levels through compression is caused by the bulk compression. When these effects are not negligible, the contribution of the bulk compression on the total turbulence level was reduced to about 85–90%. The general trend of turbulence amplification with increasing flow turning/deflection angle by the present analysis agrees well with that of others. For expansion flows, however, a quantitative comparison was challenging because the available experimental data were taken too close to the end of the perturbation and thus did not show relaxation behavior. In spite of the difficulty, the available experimental and predicted data seem to suggest that 80–85% of the reduction in turbulence level is caused by the mean density decrease through the expansion. Thus, the bulk dilation is the dominant factor for the changed turbulence intensity in a supersonic boundary layer, which experiences compression or expansion.

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